

Observation of Plasma Wave Echoes

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(Received 22 September 1967; final manuscript received 29 January 1968)

Experimental observation of a new nonlinear plasma phenomenon, the plasma wave echo, is reported. A recent theory predicts that if a longitudinal electron plasma wave is excited at one position in a collisionless plasma and Landau damps away, and a second wave is excited at another position and also damps away, then a third wave (i.e., the echo) will spontaneously appear at a third position. The existence of various echoes, associated with various orders of the perturbation theory, is demonstrated experimentally. The echoes appear at the predicted position in the plasma. The frequency of the echo wave and the dependence of its amplitude on the amplitude of the initial wave are also in agreement with the theory. Experimental data on the saturation of echo amplitude with increasing initial wave amplitude and increasing distance are given. Data are also presented on another new, but related, phenomenon "the sheath echo."

I. INTRODUCTION

It has long been recognized that electron plasma waves can be damped, even in the absence of collisions.¹ This "Landau damping" has been the subject of extensive theoretical treatments in recent years and is now believed to play an important role in many related, but more complicated, oscillation and instability phenomena. Only recently has Landau damping been demonstrated experimentally.²⁻⁵ Landau's treatment shows that macroscopic quantities such as the electric field and the charge density are damped exponentially, but that perturbations in the electron phase space distribution $f(x, v, t)$ oscillate indefinitely. Since the electron density is given by $n_e = \int f(x, v, t) dv$, one may think of the damping as arising out of the phase mixing of various parts of the distribution function. A recent theory⁶⁻⁸ shows that the direction of the phase evolution of the perturbed distribution function can be reversed by the application of a second electric field. This results in the reappearance of a macroscopic field (i.e., the echo), after both the applied electric fields have Landau damped away. The plasma echo is related to other known echo

phenomenon⁹⁻¹² in that the decay of a macroscopic physical quantity of the system, caused by phase mixing of rapidly oscillating microscopic elements in the system, is reversed by reversing the direction of phase evolution of the microscopic elements.

Observation of plasma echoes is of fundamental interest, since it experimentally verifies the reversible nature of Landau damping. In addition, it provides an experimental test of the perturbation method used to calculate the nonlinear behavior of plasmas in a wide variety of situations.

Both temporal and spatial echoes are predicted by the theory.⁸ If the experimental situation is arranged so that the primary plasma waves damp in time, a temporal echo results at a later time. If the primary plasma waves are damped in space, a spatial echo appears many Landau damping lengths away from the transmitters. The present experiment deals with the latter case.

The basic mechanism of the spatial plasma wave echo can easily be understood. When an electric field of time dependence $e^{-2\pi i f_1 t}$ is continuously excited at the point $x = 0$ in a plasma, and Landau damps as it propagates away from its point of origin, it modulates the distribution function leaving a perturbation of the form¹

$$f_1(v) \exp [2\pi i f_1 (-t + x/v)].$$

For large x , there is no electric field associated with this perturbation, since it is a rapidly oscillating function of velocity and its integral over velocity phase mixes to zero. If an additional electric field of time dependence $e^{2\pi i f_2 t}$ is excited at $x = l$ and

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² A. Y. Wong, N. D'Angelo, and R. W. Motley, *Phys. Rev.* **133**, A436 (1964).

³ J. H. Malmberg and C. B. Wharton, *Phys. Rev. Letters* **6**, 184 (1964).

⁴ J. H. Malmberg, C. B. Wharton, and W. E. Drummond, in *Plasma Physics and Controlled Nuclear Fusion Research*, (International Atomic Energy Agency, Vienna, 1966), Vol. I, p. 485.

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Landau damps as it propagates away from its point of origin, it will modulate the unperturbed part of the distribution leaving a first-order perturbation of the form $f_2(v) \exp [2\pi i f_2(t - (x-l)/v)]$, but it will also modulate the perturbation due to the first electric field, leaving a second order perturbation of the form

$$f(v) \exp [2\pi i(f_2 - f_1)t + 2\pi i f_2(l/v) - 2\pi i(f_2 - f_1)(x/v)].$$

At the point $x = l[f_2/(f_2 - f_1)]$ a velocity integral over this perturbation does not phase mix to zero; so an electric field appears at this point in the plasma. If l is large compared with a Landau damping length and $[f_2/(f_2 - f_1)]$ is of order unity, then this third electric field appears at a position well separated from the first two fields (i.e., it is a spatial echo.)

The preceding paragraph describes second-order echoes. Higher-order echoes are also predicted by the detailed theory. In general, if an electric field of frequency f_1 is continuously excited at one point in a plasma and an electric field of frequency f_2 is continuously excited at a distance l from this point, and each Landau damps in space as it propagates away from its point of origin, then a spatial echo of frequency f_3

$$f_3 = m f_2 - n f_1, \quad (1)$$

will appear at a distance l^* ,

$$l^* = \left(\frac{n f_1}{f_3}\right) l, \quad (2)$$

from the point where the second field is excited, provided that

$$m f_2 > n f_1. \quad (3)$$

In general echoes will appear on both sides of the pair of transmitting antennas but the situation is *not* symmetric: the echoes in opposite directions will differ in frequency, position, and amplitude unless $f_1 = f_2$. The integers m and n are associated with the order of the perturbation theory predicting the particular echo and for small amplitude initial waves

$$A_3 \sim A_1^n A_2^m, \quad (4)$$

where A_1 , A_2 , and A_3 are the amplitude of the wave of frequency f_1 , the wave of frequency f_2 , and of the echo, respectively. The relative phase of the echo wave φ at a distance x from the first transmitter is given by

$$\varphi \cong c \left[l \left(1 + \frac{n f_1}{f_3} \right) - x \right], \quad (5)$$

where c is almost constant. Note that, in general, motion of one of the primary transmitters changes the phase of the received signal at a different rate than motion of the wave detector.

The echoes are associated with perturbations of the electron velocity distribution remaining after the original electric fields have Landau damped away. This fact permits a curious phenomenon: echoes from a single transmitter antenna located near a sheath bounding the plasma. Electrons headed toward the sheath pass the antenna and receive an initial velocity perturbation. They are reflected elastically by the sheath and again return to the antenna going in the other direction with just the same velocity perturbation as if there were a "virtual antenna" located behind the sheath. On the return pass they are perturbed again. Third- and higher-order echoes result.

All the properties of spatial plasma wave echoes described in the previous two paragraphs have been measured and found to agree with the theory. Echoes are expected and observed for both the "lower branch" waves near the plasma frequency and the "upper branch" waves¹³⁻¹⁴ near the electron cyclotron frequency. The results reported here deal with the former case.

These experiments were performed with an apparatus¹⁵ previously used to test the Landau damping and dispersion theory¹ experimentally.³⁻⁵ Since the echo theory is essentially a second- and higher-order perturbation theory compared with the first-order perturbation theory of Landau, a prior test of the Landau theory is a logical necessity. From an experimental point of view, the prior detailed, high precision check of the Landau damping and temperature-dominated dispersion of the plasma waves is necessary to guarantee that the plasma meets the assumptions of the echo theory, i.e., that the plasma is "collisionless" in the sense of the theory, that the electron velocity distribution is Maxwellian, that the waves are properly identified, etc. The experimental plasma is, of

¹³ A. W. Trivelpiece and R. W. Gould, *J. Appl. Phys.* **30**, 1784 (1959).

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¹⁵ J. H. Malmberg, N. W. Carlson, C. B. Wharton, and W. E. Drummond, in *Proceedings of the Sixth International Conference on Ionization Phenomena in Gases*, T. Hubert and E. Crémien, Eds. (Bureau des Editions, Centre d'Etudes Nucléaires de Saday, Paris, 1964), Vol. 4, p. 229.

course, of finite radial extent, a circumstance not yet included in the theory. We do not expect this factor to change the position, frequency or amplitude dependence of the echoes. Absolute amplitudes are probably affected quantitatively. The experimental techniques used for the present experiment are almost identical to those used in prior work.³⁻⁵ In Sec. II we describe the apparatus and methods of measurement. In Sec. III the experimental results on the plasma wave echoes are presented. In Sec. IV the sheath echo is described. Our conclusions are summarized in Sec. V.

II. EXPERIMENTAL METHOD

The experimental geometry is a long column of plasma bounded in the radial direction by a good conductor. The plasma is immersed in a uniform magnetic field parallel to the axis of the plasma column. The plasma density is a function of radius, but its temperature is not. This steady-state plasma is produced in a duoplasmatron-type hydrogen-arc source and drifts from it through a cusp magnetic field into a long uniform magnetic field of a few hundred gauss. The plasma column is terminated at the end opposite to the source by a negatively charged plate. The electric field owing to this plate reflects the plasma electrons. Plasma ions strike the plate and recombine.

For the data reported here, the resulting cylindrical plasma column has a length of 180 cm and a central density of 1.5×10^8 electrons/cm³. It is immersed in a magnetic field of 305 G, which is large enough to be considered "infinite" in the sense of the theory to be tested. The background pressure is 1.5×10^{-5} Torr (mostly H₂). The plasma temperature is 9.4 eV; hence, the Debye length is about 2 mm. The electron mean free path for electron-ion collisions is of the order of 1000 m and for electron-neutral collisions is about 40 m. The plasma is surrounded by a stainless-steel tube 5.2 cm in radius, which acts as a waveguide beyond cutoff to reduce electromagnetic coupling between probes.

Three radial probes are used as antennas. Each probe consists of an 0.07-mm-diam tungsten wire exposed 1 mm at the end, a glass insulating sleeve, a coaxial stainless steel tube, and a second glass sleeve on the outside. The dimensions give a 50 Ω impedance so that each probe is a matched, insulated extension of the rf driving cables. The small over-all diameter of 0.5 mm gives minimum perturbations in the plasma. The probes may be moved the full length of the plasma in a slot in the surrounding tube. The slot has a contacting closure

which opens as the probe approaches and closes behind it. The probes can also be moved radially and rocked in angle to position them in the center of the plasma. A detector is built into the base of the probes to monitor the rf voltage level when they are used as transmitters. Two of the probes are connected by coaxial cable to transmitters which provide a rf signal of variable power and frequency. The transmitter power may be chopped at 2500 cycles when coherent detection is used. The third probe is connected to a tuned receiver whose output drives a coherent detector operated at the transmitter chopping frequency. Provision is made to add a reference signal from the transmitter to the receiver rf signal; i.e., we may use the system as an interferometer. The reference signals are not chopped since this would create a large base-line offset in the output of the coherent detector. For some experiments we require an interferometer operating at some combination of the transmitter frequencies [Eq. (1)]. In such cases we apply both reference signals to a diode and pass the output through a narrow band amplifier set at the appropriate frequency to provide the reference signal.

To obtain the dispersion relation and damping of the plasma waves, a transmitter is set at a series of fixed frequencies, and at each, the receiving probe is moved longitudinally. The position of the receiving probe is transduced to the x axis of an x - y recorder, and the output of the coherent detector is applied to the y axis. The final output is proportional to input rf power when no reference is used and to input rf voltage for the interferometer case.

From the wavelengths measured with the interferometer and the transmitter frequencies we plot the dispersion data for the waves, Fig. 1. The curve is computed from theory.⁵ The difference in dispersion curves for transmission in opposite directions is small. In the region $k < 1$ cm⁻¹ the dispersion is dominated by finite size effects. In the region $k > 1$ cm⁻¹, the dispersion is dominated by finite temperature effects. In the latter region we obtain an almost "infinite-plasma" result. The observed waves are the lowest angular and radial eigenmode of the system. There is a double infinity of waves corresponding to various radial and angular eigenmodes. However, all higher modes at a given frequency are very heavily damped compared with the lowest mode, i.e., that with angular symmetry and the simplest radial dependence. Hence, when we apply a given frequency to the transmitting antenna, only the lowest mode is observable a short

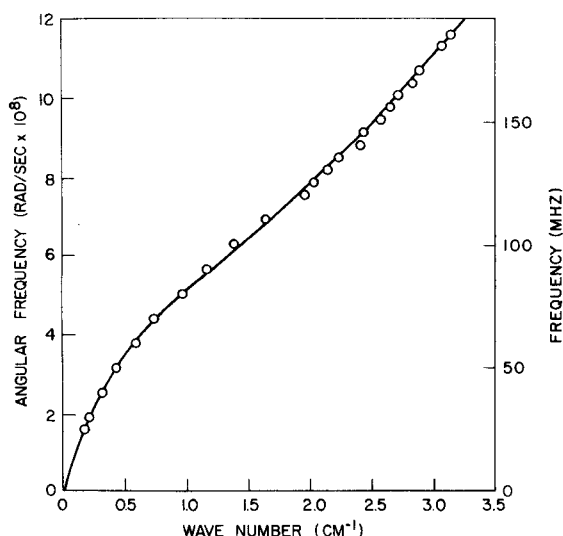


FIG. 1. Plasma wave dispersion curve. Circles are experimental result. The curve is theoretical.

distance away, and only its properties are measured. The Landau damping of the waves is measured by plotting the logarithm of the received power versus position as the receiver probe is moved. The slope of this curve gives the damping length. Finite size effects are not important for the wave damping: the result is always essentially the same in a finite plasma as for waves of the same phase velocity in an infinite plasma.⁴ Routine Landau damping measurements on the particular plasma used for the present experiment produced the theoretically predicted dependence on wave phase velocity and rough agreement in absolute magnitude. High precision damping measurements were not attempted. The plasma density varies by $\pm 6\%$ over the length of the plasma used for these experiments, producing a corresponding variation of a few percent in wavenumber at a fixed frequency from one end of the plasma to the other. We can accurately correct for this effect for wavenumber comparisons for which it may matter.

III. ECHO MEASUREMENTS

A plasma wave echo is exhibited in Fig. 2. One transmitter probe is positioned at 0 cm and a chopped 120 MHz signal applied. The second is positioned at 40 cm and an unchopped 130 MHz signal applied. The echo frequency, 140 MHz, corresponds to $m = 2$, $n = 1$, [Eq. (1)] and is thus a third-order echo. The receiver is tuned successively to 120, 130, and 140 MHz and the appropriate reference signal provided for each case. At 140 MHz, the receiver sensitivity is also increased by 20 dB.

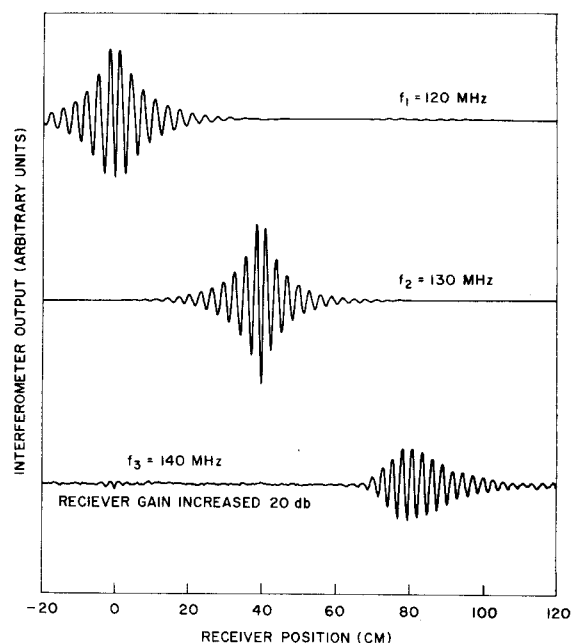


FIG. 2. Third-order echo. The transmitter probes are at 0 and 40 cm. Upper curve, receiver tuned to f_1 ; second curve, receiver tuned to f_2 ; third curve receiver tuned to f_3 and gain increased 20 dB.

The three curves are tracings of the interferometer output obtained at each frequency as the receiver probe is moved the length of the plasma. The three waves are separated in frequency as well as in space. The primary signals are rejected by the tuned filter in the receiver chain, and by having the wrong frequency for the interferometer so that the separation of the echo signal is almost perfect even though it is much smaller. In addition f_2 was not chopped when the echo curve was taken and so was rejected by the coherent detector; f_1 , and correspondingly the echo, were chopped.

The two primary, Landau damped waves are launched in both directions. The near field signal of the probes has been almost completely removed by proper adjustment of the phase of the interferometer. Figure 2 demonstrates our most important result: that plasma wave echoes exist. We have frequency-analyzed the echo signal and obtain a full width at half maximum of about 200 KHz. Some frequency spreading is always introduced by fluctuations in the plasma and this result is typical for a primary wave at the same frequency. By scanning the probes radially we measure the radial eigenfunction of the echo. It has a full width at half-maximum of 0.9 cm, identical to that of a primary wave at the same frequency. The wavenumber of the echo wave is 2.35 cm^{-1} , identical to that of a primary

wave of the same frequency. By analogy with the second-order spatial echo theory we expect the wave-number to be $(f_3/f_1)k_1$ on the rising part of the echo and k_3 on the decaying side. However for the dispersion given by Fig. 1, this effect, if it exists, is too small to be observed in the present experiment.

We have further tested Eqs. (1)–(3) by searching for echoes arising from various combinations of m and n and by using a wide assortment of frequencies. Echoes are routinely found for any set of frequencies in the range of the measured dispersion curve, typically with signal to noise ratios of 20–100. We have clearly identified echoes corresponding to second, third, fourth, and fifth order in the perturbation theory (i.e., $m + n = 2, 3, 4, 5$). Higher-order echoes are not lost in receiver noise but are confused by other nonlinear effects in the plasma. We have carefully searched for echoes on the “wrong side” of the transmitter pair with negative results. Equation (3) must be satisfied or no echo is observed.

A more precise check of Eq. (2) is obtained by removing the interferometer reference path and measuring the envelope of the echo power. A typical result is given in Fig. 3. (We have exhibited a second-order echo for the sake of variety.) Using similar data, we plot the position of the peak of the third-order echo against the separation of the two transmitters for various values of f_1/f_3 , the result is given in Fig. 4. The numbers on the curves are frequencies in MHz given in the order f_1, f_2 , and f_3 . The slopes of the solid lines are computed from Eq. (2). The intercepts of the theoretical curves have been chosen to best fit the data. Since the apparent “position” of the antennas is localized no better than a damping length, the nonzero intercepts do not indicate a discrepancy with theory. The most distinctive characteristic of plasma wave echoes

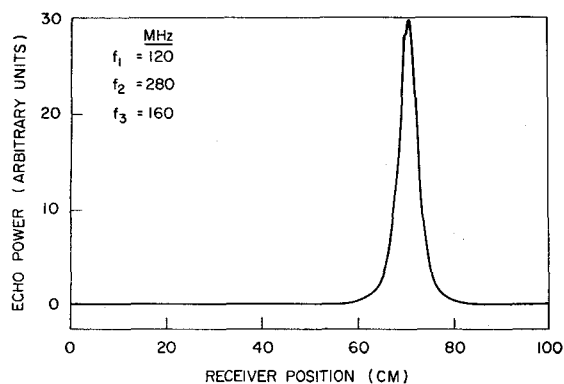


FIG. 3. Second-order echo. $f_1 = 120$ MHz, 0 cm. $f_2 = 280$ MHz, 40 cm. $f_3 = 160$ MHz.

compared to other nonlinear plasma phenomena is the change in their spatial position as the transmitter separation is varied. The data of Fig. 4 are conclusive proof of the existence of the echoes.

The amplitude of each primary wave is proportional to the rf voltage applied to its transmitter probe. This voltage is measured by a rf power meter connected to a detector built into the base of the probes. The receiver has been calibrated with a series of fixed attenuators. Using data such as Fig. 3 we measure the dependence of peak echo power on the power of the primary waves. The results for a third-order echo are shown in Fig. 5. For small signals P_3 is proportional to P_1 and to P_2^2 , and thus $A_3 \sim A_1 A_2^2$ as expected from Eq. (4) since $n = 1$ and $m = 2$ for this case. The lines drawn in Fig. 5 have the slopes predicted by Eq. (4). At large signal levels, the echo amplitude saturates. This is expected since the perturbation treatment of the problem must be carried to higher order when the signals are this large. The absolute power levels in the figures are only approximate since the probe-plasma coupling is not precisely known. The coupling was estimated by determining the total loss through the probe-plasma system for a lightly damped transmitted wave. The perturbation theory does not predict the saturation amplitude. Very roughly, we expect that the perturbation treatment leading to Eq. (4) will fail when

$$\left(\frac{e\varphi}{mv_p^2}\right)\left(\frac{k_r}{k_i}\right)(lk_r) \sim 1, \quad (6)$$

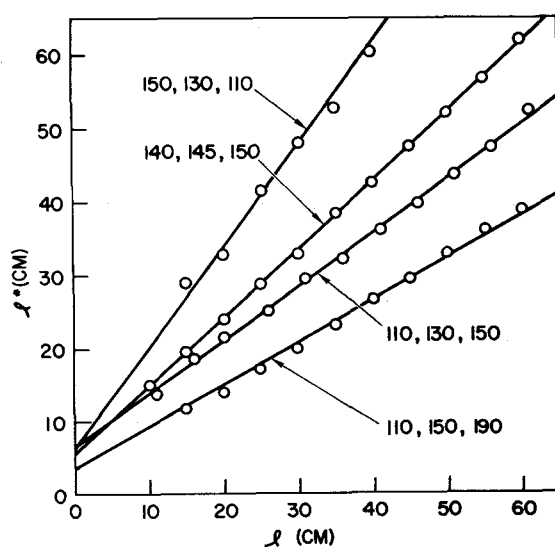


FIG. 4. Echo position versus transmitter separation. The slope of the curves is theory [Eq. (2)]. The numbers on the curves are f_1, f_2 , and f_3 in MHz.

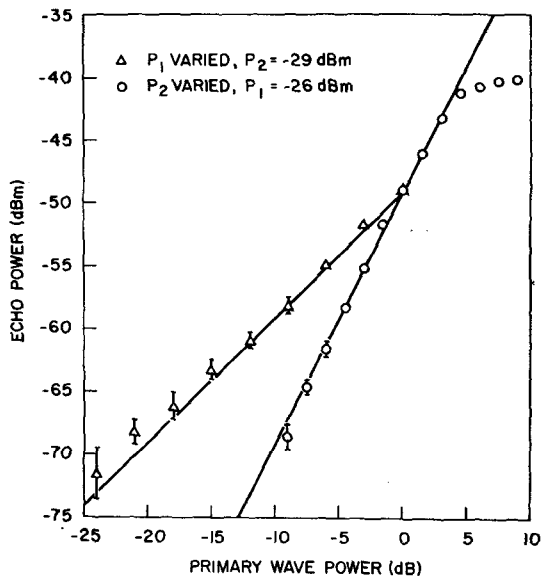


FIG. 5. Echo power versus primary wave amplitudes. Absolute power levels are approximate.

where e and m are the electron charge and mass, respectively, φ is the wave potential, v_p is the phase velocity of the wave, and k_r and k_i are the real and imaginary parts of the wavenumber, respectively. This is, roughly, the level at which saturation becomes apparent in Fig. 5.

Figure 6 gives the power at the peak of a third-order echo ($f_1 = 140$ MHz, $f_2 = 145$ MHz, $f_3 = 150$ MHz) as a function of the separation of the transmitters. The peak power rises to a maximum and then decreases as the separation is increased.

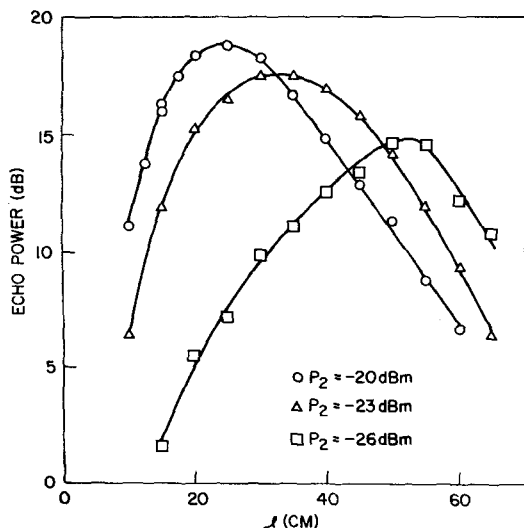


FIG. 6. Echo power versus transmitter separation. $P_1 = -32$ dBm.

The rise is well-represented by

$$A_3 = dl^3, \quad (7)$$

where d is a constant. The maximum becomes larger and occurs at smaller l as the transmitter power is increased. A maximum is expected both from large amplitude effects (saturation) and from the effects of collisions which destroy the phase information necessary for the echo. The theory is not yet sufficiently detailed to analyze the data of Fig. 6. By analogy with other echo phenomenon we expect data like Fig. 6 might ultimately be reduced to provide information on collisional processes in the plasma.

The growth and decay lengths of second-order echoes in an infinite plasma are predicted by the theory.⁸ Such a detailed theory for higher-order echoes is not yet available. The present measurements do not provide a reliable shape measurement for the third-order echoes, which we have studied experimentally in the most detail. It is known from previous Landau damping measurements⁴ that if the receiver bandwidth is too narrow, the true damping lengths are not obtained and most of the present measurements were made with a very small receiver bandwidth. Thus we have neither the theory nor the data to make detailed shape comparison. Most of the data on third-order echoes indicate the decay of the signal is slower than the rise, even for cases where by analogy with the second-order theory the reverse is expected.

IV. SHEATH ECHO

Third- and higher-order echoes may be obtained with only one transmitter probe if it is near the sheath at the end of the machine, since as previously explained, the reflection in the sheath provides a "virtual transmitter," equally spaced behind the sheath. This kind of echo is shown in Fig. 7. The single transmitter probe is approximately 30 cm

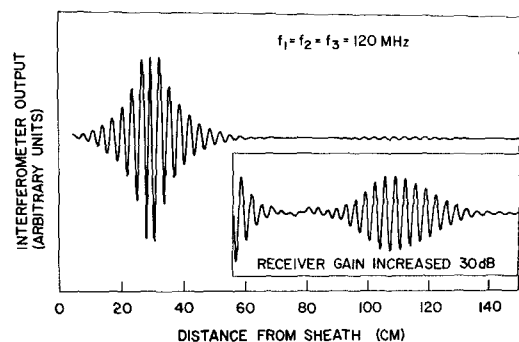


FIG. 7. Sheath echo. Receiver gain increased 30 dB for lower curve.

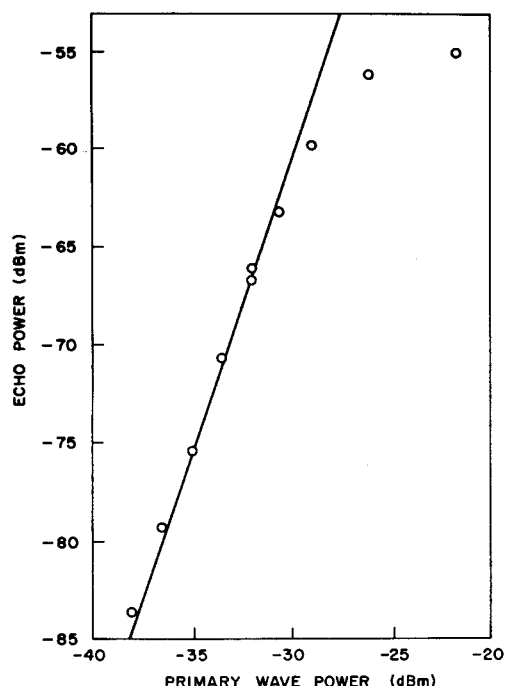


FIG. 8. Sheath echo power versus transmitter power.

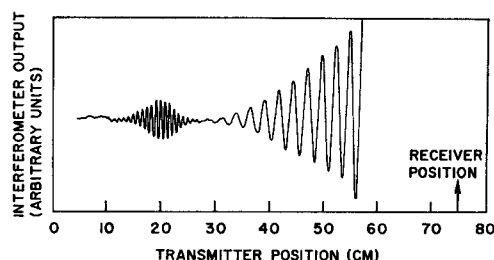


FIG. 9. Sheath echo. Receiver fixed, moving transmitter.

transmitter are moving, the *apparent* wavelength of the echo is exactly one-third that of the primary wave. This method of taking data is extremely useful, particularly during initial surveys, since it is possible to identify the echo at a glance from its anomalously short *apparent* wavelength.

We have also made extensive observations of sheath echoes with waves near the electron cyclotron frequency. These cyclotron wave echoes are not more difficult to observe than those from lower branch waves, but are much more difficult to discuss since the primary waves are not understood as well. These results will not be given here.

V. CONCLUSIONS

The existence of various spatial echoes, associated with second-, third-, fourth-, and fifth-order perturbation theory, has been demonstrated experimentally. The echoes appear at the predicted position in the plasma. The frequency and wavelength of the echo wave and the dependence of its amplitude on the amplitude of the initial waves are in agreement with theory. The existence of the "sheath echo" is also experimentally established.

Observation of plasma echoes experimentally verifies the reversible nature of collisionless damping. In addition, it provides an experimental test of the perturbation method used to predict the nonlinear behavior of plasmas in a wide variety of situations. Further refinement of these experiments could lead to a tool for studying collisional processes in plasmas.

ACKNOWLEDGMENTS

We thank C. D. Moore and P. J. Vidmar for assistance with the instrumentation and measurements.

This research was sponsored by the Defense Atomic Support Agency under contract DA-49-146-XZ-486.

from the sheath near the end plate of the machine which reflects the electrons. The effective transmitter separation is 60 cm and an echo appears at approximately that distance from the probe. The echo is large enough to be clearly visible on the same scale as the primary wave. For the inset, the receiver gain was increased 30 dB. We have measured the position of this echo as a function of transmitter position to obtain a positive identification. We have also measured the echo wave power as a function of primary wave power. The result is Fig. 8. The slope of the theoretical curve is determined from

$$A_3 \sim A_1^3. \quad (8)$$

The experimental echo amplitude is cubic in the primary wave amplitude below the saturation level, as expected for a third-order echo.

We test Eq. (5) by leaving the receiver fixed and moving a transmitter. If the transmitter position is transduced to the x axis of the recorder and the interferometer output transduced to the y axis, an *apparent* wavelength is obtained which may be used to deduce whether the relative phase of the echo moves at the same rate as the maximum amplitude of the echo. The answer is, it does. Such an interferometer curve is shown in Fig. 9 for a sheath echo. Since in this case, both the real and the virtual